Algorithms for Classical Planning

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Planning

What to do to achieve your objectives?

- Which actions to take to achieve your objectives?
- Number of agents
  - single agent, perfect information: s-t-reachability in succinct graphs
  - + nondeterminism/adversary: and-or tree search
  - + partial observability: and-or search in the space of beliefs
- Time
  - asynchronous or instantaneous actions (integer time, unit duration)
  - rational/real time, concurrency
- Objective
  - Reach a goal state.
  - Maximize probability of reaching a goal state.
  - Maximize (expected) rewards.
  - temporal goals (e.g. LTL)

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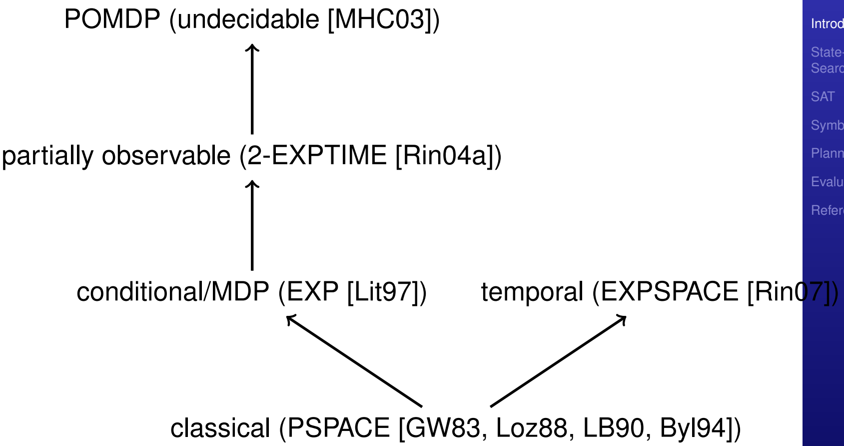
Hierarchy of Planning Problems



POMDP (undecidable [MHC03])

partially observable (2-EXPTIME [Rin04a])

conditional/MDP (EXP [Lit97])

temporal (EXPSPACE [Rin07])

classical (PSPACE [GW83, Loz88, LB90, Byl94])

Classical (Deterministic, Sequential) Planning

- states and actions expressed in terms of state variables
- single initial state, that is known
- all actions deterministic
- actions taken sequentially, one at a time
- a goal state (expressed as a formula) reached in the end

Deciding whether a plan exists is PSPACE-complete.
With a polynomial bound on plan length, NP-complete.

Domain-Independent Planning

What is domain-independent?

- general language for representing problems (e.g. PDDL)
- general algorithms to solve problems expressed in it

Advantages and disadvantages:

+ Representation of problems at a high level
+ Fast prototyping
+ Often easy to modify and extend
- Potentially high performance penalty w.r.t. specialized algorithms
- Trade-off between generality and efficiency

Domain-Specific Planning

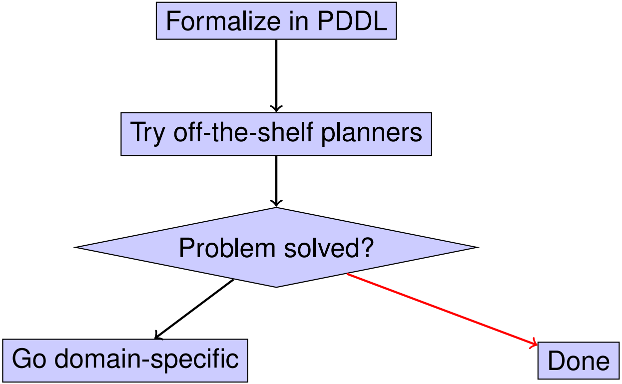
What is domain-specific?

- application-specific representation
- application-specific constraints/propagators
- application-specific heuristics

There are some planning systems that have aspects of these, but mostly this means: implement everything from scratch.

Domain-Dependent vs. -Independent Planning

Procedure



Formalize in PDDL

Try off-the-shelf planners

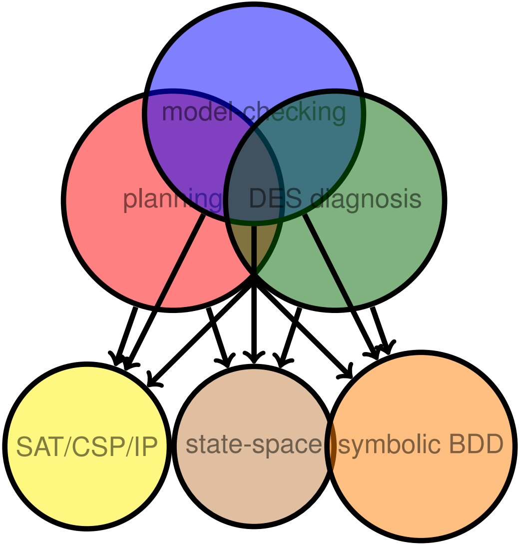
Problem solved?

Go domain-specific

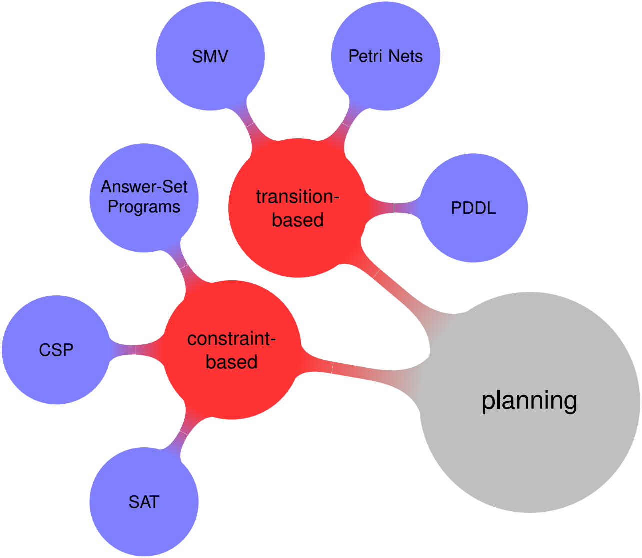
Done

Related Problems, Reductions

planning, diagnosis [SSL^{+}95], model-checking (verification)



How to Represent Planning Problems?



PDDL - Planning Domain Description Language

- Defined in 1998 [McD98], with several extensions later.
- Lisp-style syntax
- Widely used in the planning community.
- Most basic version with Boolean state variables only.
- Action sets expressed as schemata instantiated with objects.

```
(:action analyze-2
   :parameters (?s1 ?s2 - segment ?c1 ?c2 - car)
   :precondition (and (CYCLE-2-WITH-ANALYSIS ?s1 ?s2)
                      (on ?c1 ?s1))
   :effect (and (not (on ?c1 ?s1))
                (on ?c2 ?s1)
                (analyzed ?c1)
                (increase (total-cost) 3)))
```

States

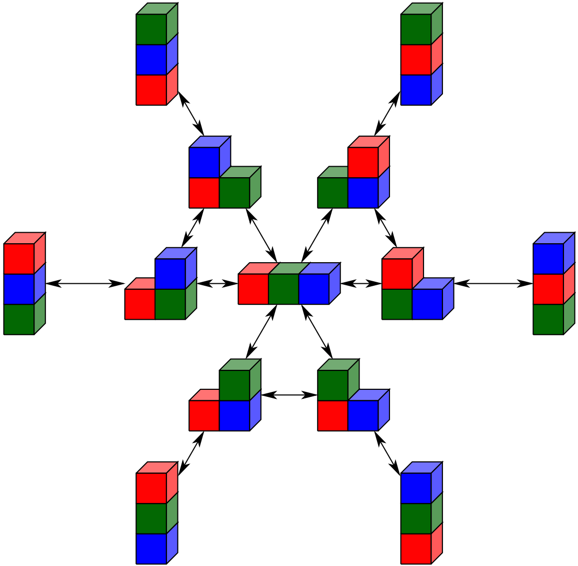
States are valuations of state variables.

Example

State variables are		One state is	
LOCATION:	$\{0, \ldots, 1000\}$	LOCATION =	312
GEAR:	$\{R, 1, 2, 3, 4, 5\}$	GEAR =	4
FUEL:	$\{0, \ldots, 60\}$	FUEL =	58
SPEED:	$\{-20, \ldots, 200\}$	SPEED =	110
DIRECTION:	$\{0, \ldots, 359\}$	DIRECTION =	90

State-space transition graphs

Blocks world with three blocks



Actions

How values of state variables change

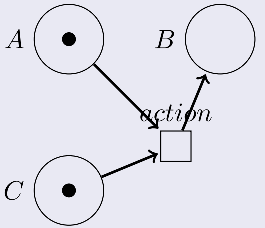
General form

precondition: A=1 $\wedge$ C=1
effect: A := 0; B := 1; C := 0;

STRIPS representation

PRE: A, C
ADD: B
DEL: A, C

Petri net



Weaknesses in Existing Languages

- High-level concepts not easily/efficiently expressible. Examples: graph connectivity, transitive closure.
- Limited or no facilities to express domain-specific information (control, pruning, heuristics).
- The notion of classical planning is limited:
  - Real world rarely a single run of the sense-plan-act cycle.
  - Main issue often uncertainty, costs, or both.
  - Often rational time and concurrency are critical.

Formalization of Planning in This Tutorial

A problem instance in (classical) planning consists of the following.

- set X of state variables
- set A of actions $\langle p, e \rangle$ where
  - p is the precondition (a set of literals over X)
  - e is the effects (a set of literals over X)
- initial state $I : X \to \{0, 1\}$ (a valuation of X)
- goals G (a set of literals over X)

The planning problem

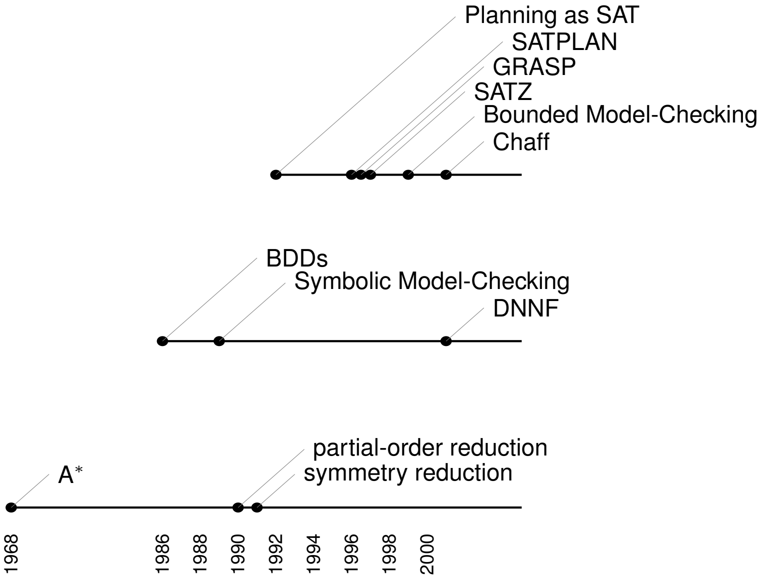
An action $a = \langle p, e \rangle$ is applicable in state s iff $s \models p$.
The successor state $s' = \mathit{exec}_a(s)$ is defined by

- $s' \models e$
- $s(x) = s'(x)$ for all $x \in X$ that don't occur in e.

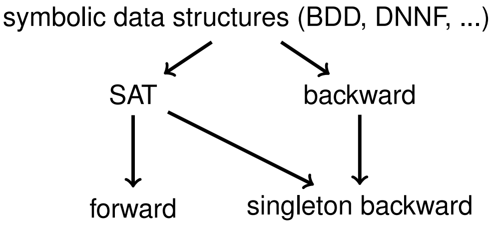
Problem

Find $a_1, \dots, a_n$ such that
$\mathit{exec}_{a_n}(\mathit{exec}_{a_{n-1}}(\cdots \mathit{exec}_{a_2}(\mathit{exec}_{a_1}(I)) \cdots)) \models G$?

Development of state-space search methods



Symbolic Representations vs. Fwd and Bwd Search



symbolic data structures (BDD, DNNF, ...)

SAT

backward

forward

singleton backward

1. symbolic data structures
2. SAT
3. state-space search
4. others: partial-order planning [MR91] (until 1995)

Explicit State-Space Search

- The most basic search method for transition systems
- Very efficient for small state spaces (1 million states)
- Easy to implement
- Very well understood
- Pruning methods:
    - symmetry reduction [Sta91, ES96]
    - partial-order reduction [God91, Val91]
    - lower-bounds / heuristics, for informed search [HNR68]

State Representation

Each state represented explicitly ⇒ compact state representation important

- Boolean (0, 1) state variables represented by one bit
- Inter-variable dependencies enable further compaction:
  - ¬(at(A,L1)∧at(A,L2)) always true
  - automatic recognition of invariants [BF97, Rin98, Rin08]
  - n exclusive variables $x_1, \ldots, x_n$ represented by $1 + \lfloor \log_2(n-1) \rfloor$ bits

Search Algorithms

- uninformed/blind search: depth-first, breadth-first, ...
- informed search: “best first” search (always expand best state so far)
- informed search: local search algorithms such as simulated annealing, tabu search and others [KGJV83, DS90, Glo89] (little used in planning)
- optimal algorithms: A* [HNR68], IDA* [Kor85]

Symmetry Reduction [Sta91, ES96]

Idea

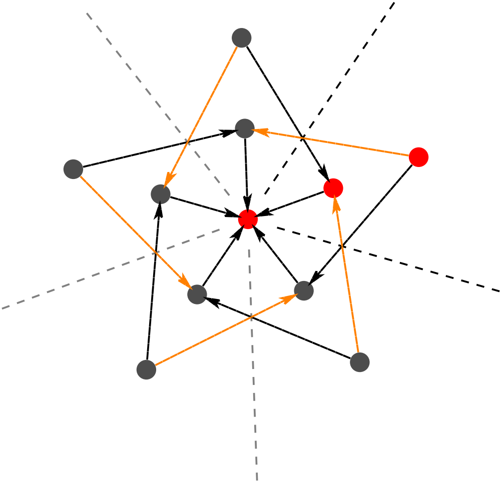
1. Define an equivalence relation $\sim$ on the set of all states: $s_1 \sim s_2$ means that state s_1 is symmetric with s_2.
2. Only one state s_C in each equivalence class C needs to be considered.
3. If state $s \in C$ with $s \neq [s_C]$ is encountered, replace it with s_C.

Example

States $P(A) \wedge \neg P(B) \wedge P(C)$ and $\neg P(A) \wedge P(B) \wedge P(C)$ are symmetric because of the permutation $A \mapsto B, B \mapsto A, C \mapsto C$.

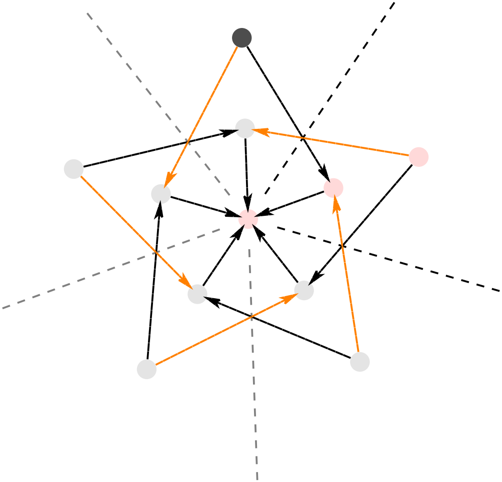
Symmetry Reduction

Example: 11 states, 3 equivalence classes



Symmetry Reduction

Example: 11 states, 3 equivalence classes



Symmetry Reduction

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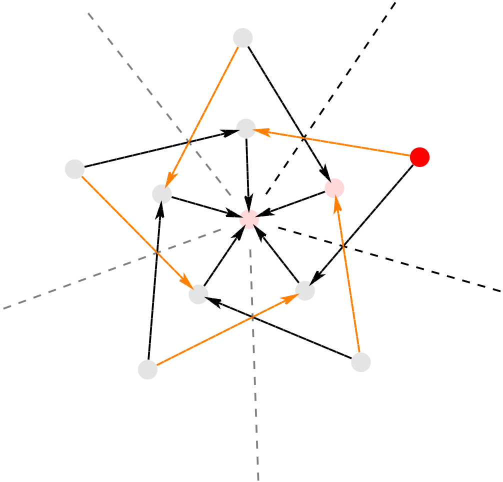
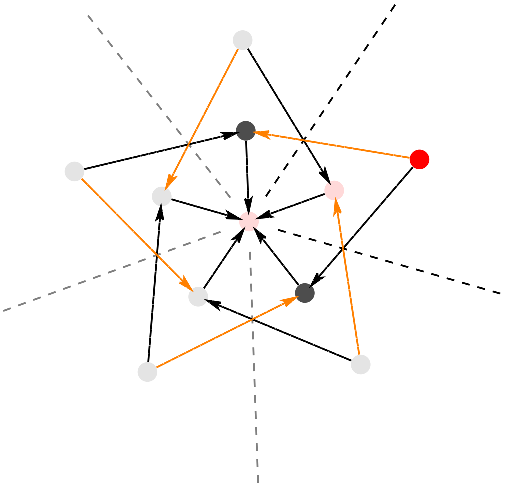


Symmetry Reduction

Example: 11 states, 3 equivalence classes



Symmetry Reduction

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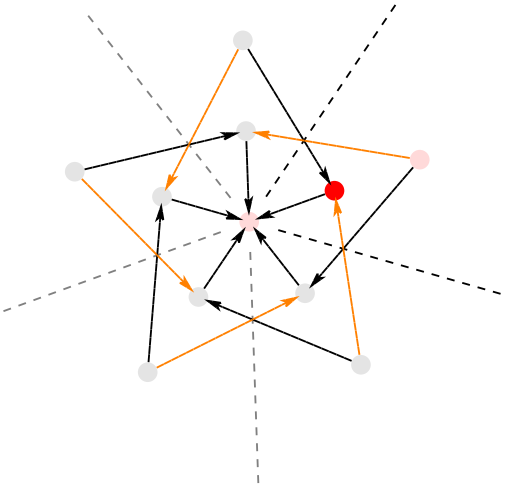

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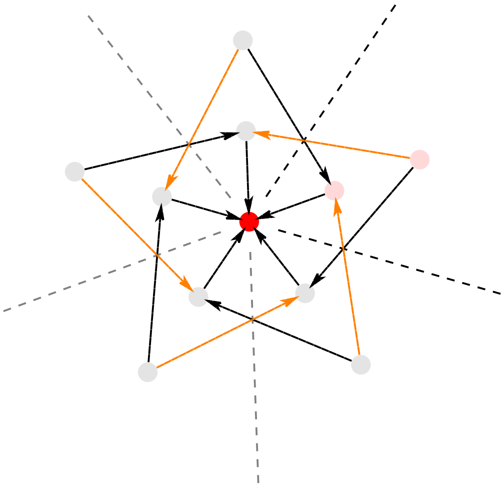
Example: 11 states, 3 equivalence classes



Partial Order Reduction

Stubborn sets and related methods

Idea [God91, Val91]

Independent actions unnecessary to consider in all orderings, e.g. both A_1, A_2 and A_2, A_1.

Example

Let there be lamps $1, 2, \ldots, n$ which can be turned on. There are no other actions. One can restrict to plans in which lamps are turned on in the ascending order: switching lamp n after lamp $m > n$ needless.[a]

---

[a] The same example is trivialized also by symmetry reduction!

Heuristics for Classical Planning

The most basic heuristics widely used for non-optimal planning:

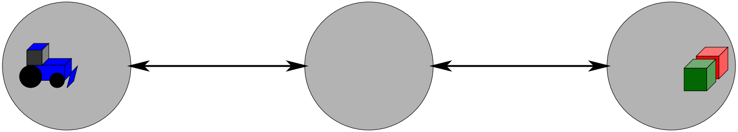
h^{max}	[BG01, McD96]	best-known admissible heuristic
h^{+}	[BG01]	still state-of-the-art
h^{relax}	[HN01]	often more accurate, but performs like h^{+}

Definition of h^{max}, h^{+} and h^{relax}

- Basic insight: estimate distances between possible state variable values, not states themselves.
- $g_s(l) = \begin{cases} 0 & \text{if } s \models l \\ \min_{a \text{ with effect } p} (1 + g_s(\text{prec}(a))) \end{cases}$
- h^{+} defines $g_s(L) = \sum_{l \in L} g_s(l)$ for sets S.
- h^{max} defines $g_s(L) = \max_{l \in L} g_s(l)$ for sets S.
- h^{relax} counts the number of actions in computation of h^{max}.

Computation of h^{max}

Tractor example



1. Tractor moves:
   - from 1 to 2: $T12 = \langle T1, \{T2, \neg T1\}\rangle$
   - from 2 to 1: $T21 = \langle T2, \{T1, \neg T2\}\rangle$
   - from 2 to 3: $T23 = \langle T2, \{T3, \neg T2\}\rangle$
   - from 3 to 2: $T32 = \langle T3, \{T2, \neg T3\}\rangle$
2. Tractor pushes A:
   - from 2 to 1: $A21 = \langle T2 \wedge A2, \{T1, A1, \neg T2, \neg A2\}\rangle$
   - from 3 to 2: $A32 = \langle T3 \wedge A3, \{T2, A2, \neg T3, \neg A3\}\rangle$
3. Tractor pushes B:
   - from 2 to 1: $B21 = \langle T2 \wedge B2, \{T1, B1, \neg T2, \neg B2\}\rangle$
   - from 3 to 2: $B32 = \langle T3 \wedge B3, \{T2, B2, \neg T3, \neg B3\}\rangle$

Computation of h^{max}
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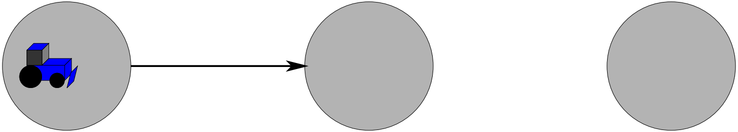
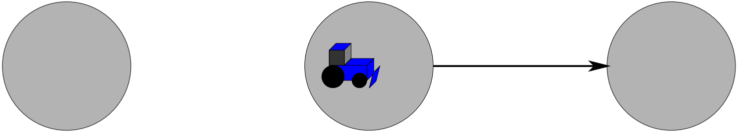


t	T1	T2	T3	A1	A2	A3	B1	B2	B3
0	T	F	F	F	F	T	F	F	T
1	TF	TF	F	F	F	T	F	F	T
2	TF	TF	TF	F	F	T	F	F	T
3	TF	TF	TF	F	TF	TF	F	TF	TF
4	TF	TF	TF	TF	TF	TF	TF	TF	TF

Apply $T12 = \langle T1, \{T2, \neg T1\} \rangle$

Computation of h^{max}

Tractor example



t	T1	T2	T3	A1	A2	A3	B1	B2	B3
0	T	F	F	F	F	T	F	F	T
1	TF	TF	F	F	F	T	F	F	T
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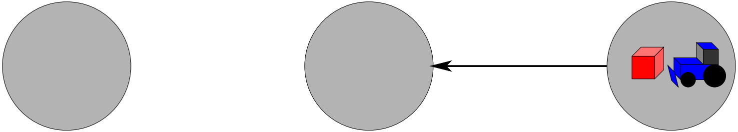
3	TF	TF	TF	F	TF	TF	F	TF	TF
4	TF	TF	TF	TF	TF	TF	TF	TF	TF

Apply $T23 = \langle T2, \{T3, \neg T2\}\rangle$

Computation of h^{max}

Tractor example



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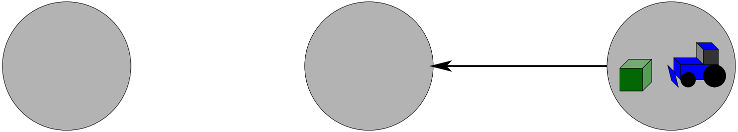
2	TF	TF	TF	F	F	T	F	F	T
3	TF	TF	TF	F	TF	TF	F	TF	TF
4	TF	TF	TF	TF	TF	TF	TF	TF	TF

Apply $A32 = \langle T3 \wedge A3, \{T2, A2, \neg T3, \neg A3\}\rangle$

Computation of h^{max}

Tractor example



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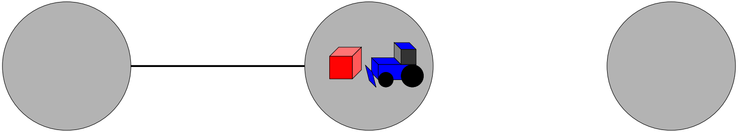
Apply $B32 = \langle T3 \wedge B3, \{T2, B2, \neg T3, \neg B3\}\rangle$

Computation of h^{max}
Tractor example



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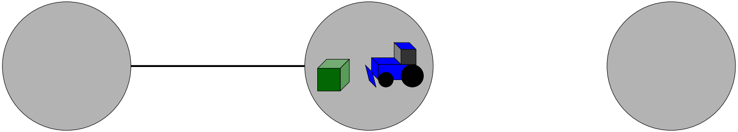
2	TF	TF	TF	F	F	T	F	F	T
3	TF	TF	TF	F	TF	TF	F	TF	TF
4	TF	TF	TF	TF	TF	TF	TF	TF	TF

Apply $A21 = \langle T2 \wedge A2, \{T1, A1, \neg T2, \neg A2\}\rangle$

Computation of h^{max}

Tractor example



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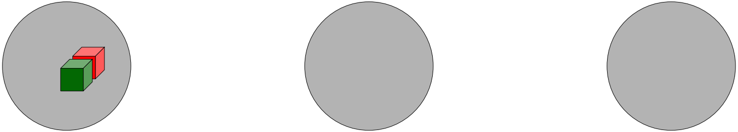
Apply $B21 = \langle T2 \wedge B2, \{T1, B1, \neg T2, \neg B2\}\rangle$

Computation of h^{max}

Tractor example



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2	TF	TF	TF	F	F	T	F	F	T
3	TF	TF	TF	F	TF	TF	F	TF	TF
4	TF	TF	TF	TF	TF	TF	TF	TF	TF

Distance of $A1 \wedge B1$ is 4.

h^{max} Underestimates

Example

Estimate for lamp1on ∧ lamp2on ∧ lamp3on with

$$\langle\top, \{\text{lamp1on}\}\rangle$$
$$\langle\top, \{\text{lamp2on}\}\rangle$$
$$\langle\top, \{\text{lamp3on}\}\rangle$$

is 1. Actual shortest plan has length 3.
By definition, $h^{max}(G_1 \wedge \cdots \wedge G_n)$ is the maximum of $h^{max}(G_1), \ldots, h^{max}(G_n)$.
If goals are independent, the sum of the estimates is more accurate.

Computation of h^+

Tractor example

t	T1	T2	T3	A1	A2	A3	B1	B2	B3
0	T	F	F	F	F	T	F	F	T
1	TF	TF	F	F	F	T	F	F	T
2	TF	TF	TF	F	F	T	F	F	T
3	TF	TF	TF	F	TF	TF	F	TF	TF
4	TF	TF	TF	F	TF	TF	F	TF	TF
5	TF	TF	TF	TF	TF	TF	TF	TF	TF

Apply $A21 = \langle T2 \wedge A2, \{T1, A1, \neg T2, \neg A2\}\rangle$.
$h^+(T2 \wedge A2)$ is 1+3.
$h^+(A1)$ is 1+3+1 = 5 (h^{max} gives 4.)

Computation of h^+

Tractor example

t	T1	T2	T3	A1	A2	A3	B1	B2	B3
0	T	F	F	F	F	T	F	F	T
1	TF	TF	F	F	F	T	F	F	T
2	TF	TF	TF	F	F	T	F	F	T
3	TF	TF	TF	F	TF	TF	F	TF	TF
4	TF	TF	TF	F	TF	TF	F	TF	TF
5	TF	TF	TF	TF	TF	TF	TF	TF	TF

h^+ of $A1 \wedge B1$ is $5 + 5 = 10$.

Computation of h^{relax}
Motivation

actions	estimate for $a \wedge b \wedge c$ max	sum	actual
$\langle \top, \{a, b, c\} \rangle$	1	3	1
$\langle \top, \{a\} \rangle, \langle \top, \{b\} \rangle, \langle \top, \{c\} \rangle$	1	3	3

- Better estimates with h^{relax} (but: performance is similar to h^{+}).
- Application: directing search with preferred actions [Vid04, RH09]

Computation of h^{relax}

t	T1	T2	T3	A1	A2	A3	B1	B2	B3
0	T	F	F	F	F	T	F	F	T
1	TF	TF	F	F	F	T	F	F	T
2	TF	TF	TF	F	F	T	F	F	T
3	TF	TF	TF	F	TF	TF	F	TF	TF
4	TF	TF	TF	TF	TF	TF	TF	TF	TF

Estimate for $A1 \wedge B1$ with relaxed plans:

t	relaxed plan
0	
1	
2	
3	

Computation of h^{relax}

t	T1	T2	T3	A1	A2	A3	B1	B2	B3
0	T	F	F	F	F	T	F	F	T
1	TF	TF	F	F	F	T	F	F	T
2	TF	TF	TF	F	F	T	F	F	T
3	TF	TF	TF	F	TF	TF	F	TF	TF
4	TF	TF	TF	TF	TF	TF	TF	TF	TF

Estimate for $A1 \wedge B1$ with relaxed plans:

t	relaxed plan
0	
1	
2	
3	A21, B21

Computation of h^{relax}

t	T1	T2	T3	A1	A2	A3	B1	B2	B3
0	T	F	F	F	F	T	F	F	T
1	TF	TF	F	F	F	T	F	F	T
2	TF	TF	TF	F	F	T	F	F	T
3	TF	TF	TF	F	TF	TF	F	TF	TF
4	TF	TF	TF	TF	TF	TF	TF	TF	TF

Estimate for $A1 \wedge B1$ with relaxed plans:

t	relaxed plan
0	
1	
2	
3	A21, B21

Computation of h^{relax}

t	T1	T2	T3	A1	A2	A3	B1	B2	B3
0	T	F	F	F	F	T	F	F	T
1	TF	TF	F	F	F	T	F	F	T
2	TF	TF	TF	F	F	T	F	F	T
3	TF	TF	TF	F	TF	TF	F	TF	TF
4	TF	TF	TF	TF	TF	TF	TF	TF	TF

Estimate for $A1 \wedge B1$ with relaxed plans:

t	relaxed plan
0	
1	
2	A32, B32
3	A21, B21

Computation of h^{relax}

t	T1	T2	T3	A1	A2	A3	B1	B2	B3
0	T	F	F	F	F	T	F	F	T
1	TF	TF	F	F	F	T	F	F	T
2	TF	TF	TF	F	F	T	F	F	T
3	TF	TF	TF	F	TF	TF	F	TF	TF
4	TF	TF	TF	TF	TF	TF	TF	TF	TF

Estimate for $A1 \wedge B1$ with relaxed plans:

t	relaxed plan
0	
1	
2	A32, B32
3	A21, B21

Computation of h^{relax}

t	T1	T2	T3	A1	A2	A3	B1	B2	B3
0	T	F	F	F	F	T	F	F	T
1	TF	TF	F	F	F	T	F	F	T
2	TF	TF	TF	F	F	T	F	F	T
3	TF	TF	TF	F	TF	TF	F	TF	TF
4	TF	TF	TF	TF	TF	TF	TF	TF	TF

Estimate for $A1 \wedge B1$ with relaxed plans:

t	relaxed plan
0	
1	T23
2	A32, B32
3	A21, B21

Computation of h^{relax}

t	T1	T2	T3	A1	A2	A3	B1	B2	B3
0	T	F	F	F	F	T	F	F	T
1	TF	TF	F	F	F	T	F	F	T
2	TF	TF	TF	F	F	T	F	F	T
3	TF	TF	TF	F	TF	TF	F	TF	TF
4	TF	TF	TF	TF	TF	TF	TF	TF	TF

Estimate for $A1 \wedge B1$ with relaxed plans:

t	relaxed plan
0	
1	T23
2	A32, B32
3	A21, B21

Computation of h^{relax}

t	T1	T2	T3	A1	A2	A3	B1	B2	B3
0	T	F	F	F	F	T	F	F	T
1	TF	TF	F	F	F	T	F	F	T
2	TF	TF	TF	F	F	T	F	F	T
3	TF	TF	TF	F	TF	TF	F	TF	TF
4	TF	TF	TF	TF	TF	TF	TF	TF	TF

Estimate for $A1 \wedge B1$ with relaxed plans:

t	relaxed plan
0	T12
1	T23
2	A32, B32
3	A21, B21

Computation of h^{relax}

t	T1	T2	T3	A1	A2	A3	B1	B2	B3
0	T	F	F	F	F	T	F	F	T
1	TF	TF	F	F	F	T	F	F	T
2	TF	TF	TF	F	F	T	F	F	T
3	TF	TF	TF	F	TF	TF	F	TF	TF
4	TF	TF	TF	TF	TF	TF	TF	TF	TF

Estimate for $A1 \wedge B1$ with relaxed plans:

t	relaxed plan
0	T12
1	T23
2	A32, B32
3	A21, B21

estimate = number of actions in relaxed plan = 6

Comparison of the Heuristics

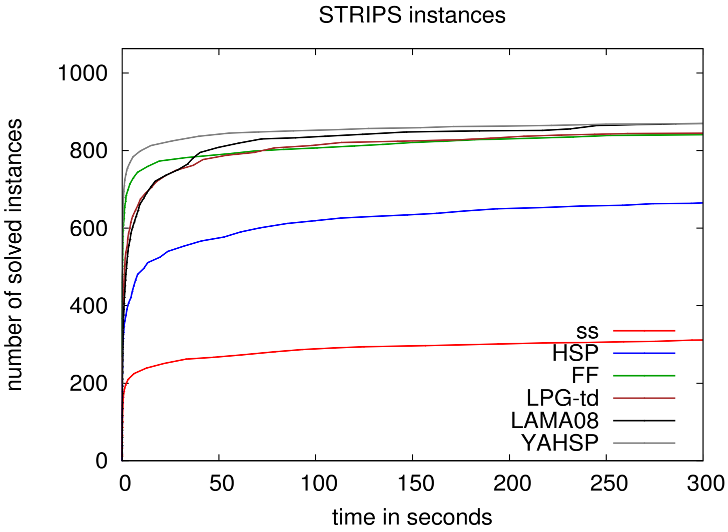
- For the Tractor example:
    - actions in the shortest plan: 8
    - h^{max} yields 4 (never overestimates).
    - h^{+} yields 10 (may under or overestimate).
    - h^{relax} yield 6 (may under or overestimate).
- The sum-heuristic and the relaxed plan heuristic are used in practice for non-optimal planners.

Preferred Actions

- h^+ and h^{relax} boosted with preferred/helpful actions.
- Preferred actions on the first level $t = 0$ in a relaxed plan.
- Several possibilities:
  - Always expand with a preferred action when possible [Vid04].
  - A tie-breaker when the heuristic values agree [RH09].
- Planners based on explicit state-space search use them: YAHSP, LAMA.

Performance of State-Space Search Planners

Planning Competition Problems



Heuristics for Optimal Planning

Admissible heuristics are needed for finding optimal plans, e.g with A* [HNR68]. Scalability much poorer.

Pattern Databases [CS96, Ede00]

Abstract away many/most state variables, and use the length/cost of the optimal solution to the remaining problem as an estimate.

Generalized Abstraction (merge and shrink) [DFP09, HHH07]

A generalization of pattern databases, allowing more complex aggregation of states (not just identification of ones agreeing on a subset of state variables.)

Landmark-cut [HD09] has been doing well with planning competition problems.

Planning with SAT
Background

- Proposed by Kautz and Selman [KS92].
- Idea as in Cook's proof of NP-hardness of SAT [Coo71]: encode each step of a plan as a propositional formula.
- Intertranslatability of NP-complete problems ⇒ reductions to many other problems possible.

Related solution methods

constraint satisfaction (CSP)	[vBC99, DK01]
NM logic programs / answer-set programs	[DNK97]

Translations from SAT into other formalisms often simple. In terms of performance, SAT is usually the best choice.

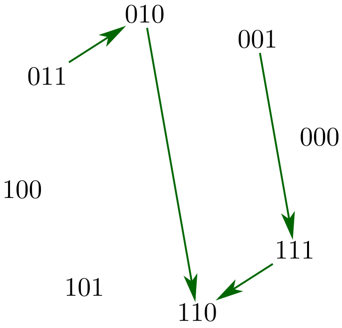
Transition relations in propositional logic

State variables are
$X = \{a, b, c\}$.

$$\begin{array}{l}
(\neg a \wedge b \wedge c \wedge \neg a' \wedge b' \wedge \neg c') \vee \\
(\neg a \wedge b \wedge \neg c \wedge a' \wedge b' \wedge \neg c') \vee \\
(\neg a \wedge \neg b \wedge c \wedge a' \wedge b' \wedge c') \vee \\
(a \wedge b \wedge c \wedge a' \wedge b' \wedge \neg c')
\end{array}$$

The corresponding matrix is

$$\begin{array}{c|cccccccc}
 & 000 & 001 & 010 & 011 & 100 & 101 & 110 & 111 \\
\hline
000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
001 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
010 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
011 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
100 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
101 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
110 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
111 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{array}$$



Encoding of Actions as Formulas
for Sequential Plans

An action j corresponds to the conjunction of the precondition $P_j@t$ and

$$x_i@(t+1) \leftrightarrow F_i(x_1@t, \ldots, x_n@t)$$

for all $i \in \{1, \ldots, n\}$. Denote this by $E_j@t$.

Example (move-from-X-to-Y)

$$\overbrace{atX@t}^{\text{precond}} \wedge \overbrace{\begin{array}{l}(atX@(t+1) \leftrightarrow \bot) \wedge (atY@(t+1) \leftrightarrow \top) \\ \wedge(atZ@(t+1) \leftrightarrow atZ@t) \wedge (atU@(t+1) \leftrightarrow atU@t)\end{array}}^{\text{effects}}$$

Choice between actions $1, \ldots, m$ expressed by the formula

$$\mathcal{R}@t = E_1@t \vee \cdots \vee E_m@t.$$

Finding a Plan with SAT

Let

- I be a formula expressing the initial state, and
- G be a formula expressing the goal states.

Then a plan of length T exists iff

$$I@0 \land \bigwedge_{t=0}^{T-1} \mathcal{R}@t \land G_T$$

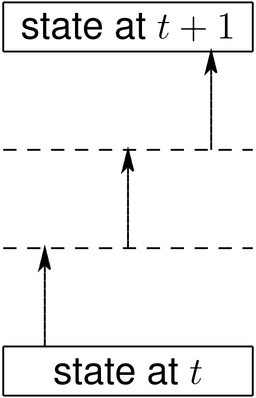
is satisfiable.

Remark

Most SAT solvers require formulas to be in CNF. There are efficient transformations to achieve this [Tse62, JS05, MV07].

Parallel Plans: Motivation

- Don't represent all intermediate states of a sequential plan.
- Ignore relative ordering of consecutive actions.
- Reduced number of explicitly represented states ⇒ smaller formulas



Parallel plans (∀-step plans)

Kautz and Selman 1996

Allow actions $a_1 = \langle p_1, e_1 \rangle$ and $a_2 = \langle p_2, e_2 \rangle$ in parallel whenever they don't interfere, i.e.

- both $p_1 \cup p_2$ and $e_1 \cup e_2$ are consistent, and
- both $e_1 \cup p_2$ and $e_2 \cup p_1$ are consistent.

Theorem

If $a_1 = \langle p_1, e_1 \rangle$ and $a_2 = \langle p_1, e_1 \rangle$ don't interfere and s is a state such that $s \models p_1$ and $s \models p_2$, then $\mathit{exec}_{a_1}(\mathit{exec}_{a_2}(s)) = \mathit{exec}_{a_2}(\mathit{exec}_{a_1}(s))$.

$\forall$-step plans: encoding

Define $\mathcal{R}^{\forall}@t$ as the conjunction of

$$x@(t+1) \leftrightarrow ((x@t \wedge \neg a_1@t \wedge \cdots \wedge \neg a_k@t) \vee a'_1@t \vee \cdots \vee a'_{k'}@t)$$

for all $x \in X$, where $a_1, \ldots, a_k$ are all actions making x false, and $a'_1, \ldots, a'_{k'}$ are all actions making x true, and

$$a@t \rightarrow l@t \text{ for all } l \text{ in the precondition of } a,$$

and

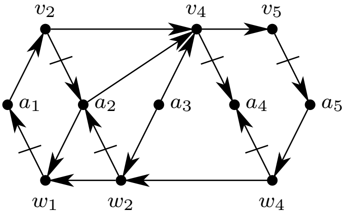
$$\neg(a@t \wedge a'@t) \text{ for all } a \text{ and } a' \text{ that interfere.}$$

This encoding is quadratic due to the interference clauses.

$\forall$-step plans: linear encoding

Rintanen et al. 2006 [RHN06]

Action a with effect l disables all actions with precondition $\bar{l}$, except a itself.
This is done in two parts: disable actions with higher index, disable actions with lower index.



This is needed for every literal.

$\exists$-step plans

Dimopoulos et al. 1997 [DNK97]

Allow actions $\{a_1, \ldots, a_n\}$ in parallel if they can be executed in at least one order.

- $\bigcup_{i=1}^{n} p_i$ is consistent.
- $\bigcup_{i=1}^{n} e_i$ is consistent.
- There is a total ordering $a_1, \ldots, a_n$ such that $e_i \cup p_j$ is consistent whenever $i \leq j$: disabling an action earlier in the ordering is allowed.

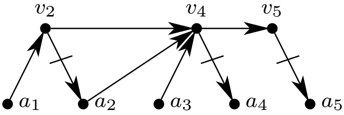
Several compact encodings exist [RHN06].
Fewer time steps are needed than with $\forall$-step plans. Sometimes only half as many.

∃-step plans: linear encoding

Rintanen et al. 2006 [RHN06]

Choose an arbitrary fixed ordering of all actions $a_1, \dots, a_n$.

Action a with effect l disables all later actions with precondition $\bar{l}$.



This is needed for every literal.

Disabling graphs

Rintanen et al. 2006 [RHN06]

Define a disabling graph with actions as nodes and with an arc from a_1 to a_2 (a_1 disables a_2) if $p_1 \cup p_2$ and $e_1 \cup e_2$ are consistent and $e_1 \cup p_2$ is inconsistent.

The test for valid execution orderings can be limited to strongly connected components (SCC) of the disabling graph.

In many structured problems all SCCs are singleton sets.
$\Longrightarrow$ No tests for validity of orderings needed during SAT solving.

Summary of Notions of Plans

plan type	reference	comment
sequential	[KS92]	one action per time point
∀-parallel	[BF97, KS96]	parallel actions independent
∃-parallel	[DNK97, RHN06]	executable in at least one order

The last two expressible in terms of the relation disables restricted to applied actions:

- ∀-parallel plans: the disables relation is empty.
- ∃-parallel plans: the disables relation is acyclic.

Search through Horizon Lengths

The planning problem is reduced to the satisfiability tests for

$$
\begin{aligned}
&\Phi_0 = I@0 \wedge G@0 \\
&\Phi_1 = I@0 \wedge \mathcal{R}@0 \wedge G@1 \\
&\Phi_2 = I@0 \wedge \mathcal{R}@0 \wedge \mathcal{R}@1 \wedge G@2 \\
&\Phi_3 = I@0 \wedge \mathcal{R}@0 \wedge \mathcal{R}@1 \wedge \mathcal{R}@2 \wedge G@3 \\
&\vdots \\
&\Phi_u = I@0 \wedge \mathcal{R}@0 \wedge \mathcal{R}@1 \wedge \cdots \mathcal{R}@(u-1) \wedge G@u
\end{aligned}
$$

where u is the maximum possible plan length.

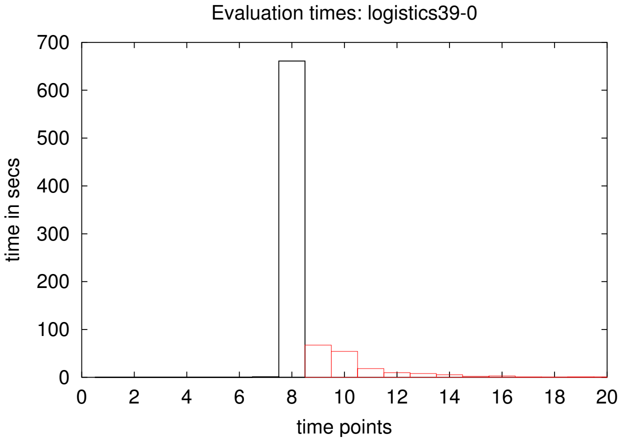
Q: How to schedule these satisfiability tests?

Search through Horizon Lengths

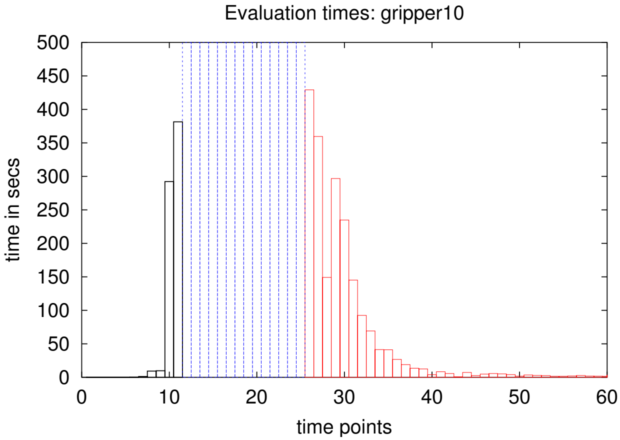
algorithm	reference	comment
sequential	[KS92, KS96]	slow, guarantees min. horizon
binary search	[SS07]	prerequisite: length UB
n processes	[Rin04b, Zar04]	fast, more memory needed
geometric	[Rin04b]	fast, more memory needed

- sequential: first test Φ_0, then Φ_1, then Φ_2, $\ldots$
  - This is breadth-first search / iterative deepening.
  - Guarantees shortest horizon length, but is slow.
- parallel strategies: solve several horizon lengths simultaneously
  - depth-first flavor
  - usually much faster
  - no guarantee of minimal horizon length

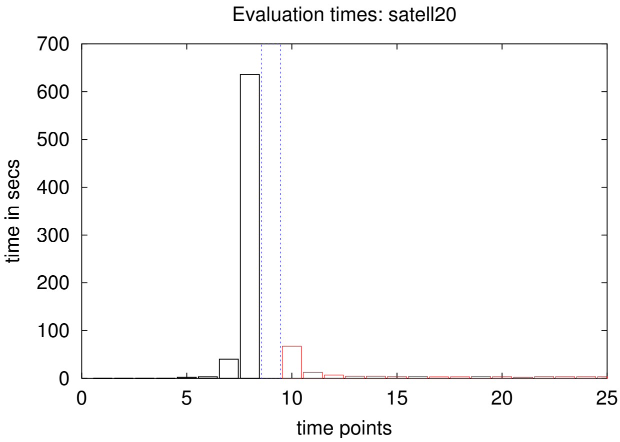
Some runtime profiles



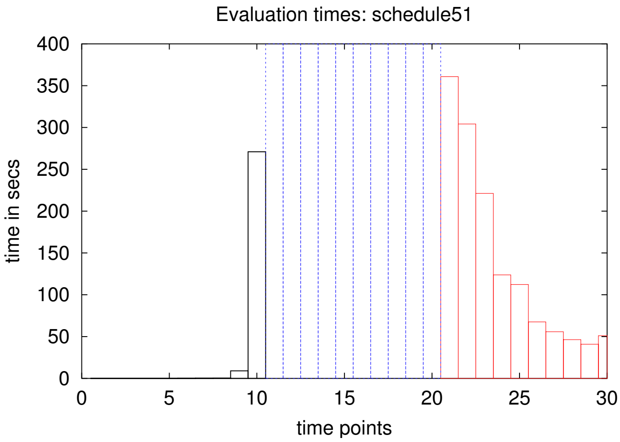
Some runtime profiles



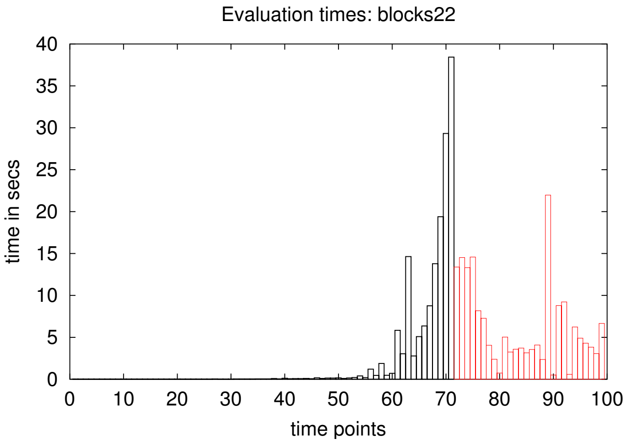
Some runtime profiles



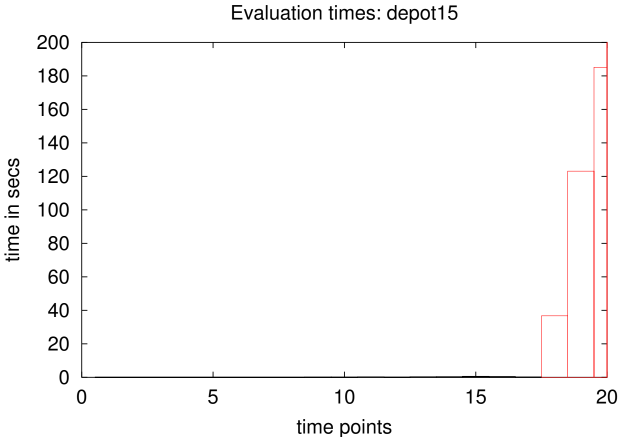
Some runtime profiles



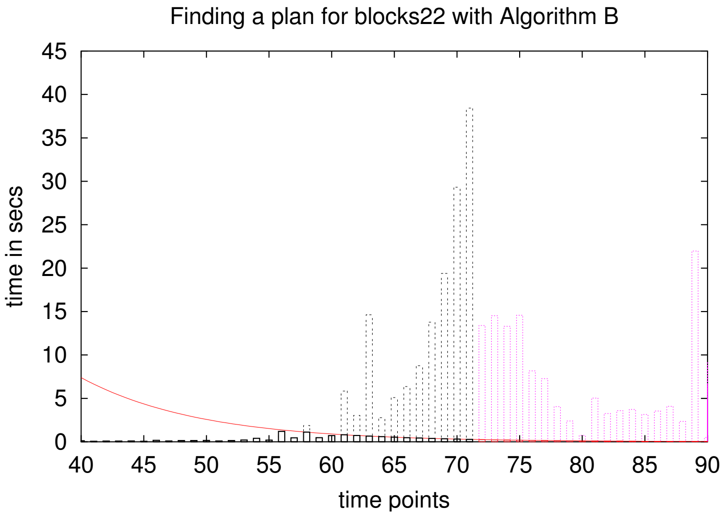
Some runtime profiles



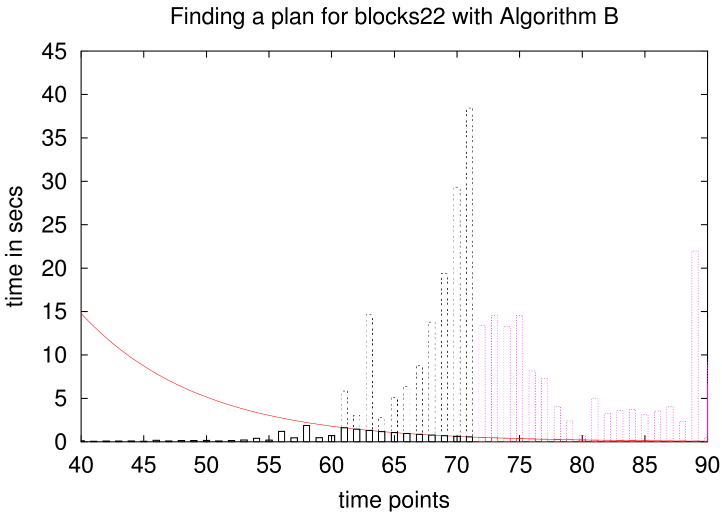
Some runtime profiles



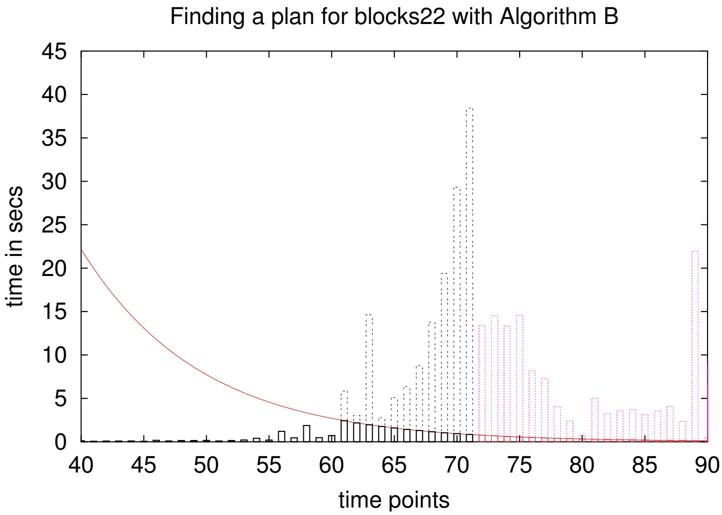
Geometric Evaluation



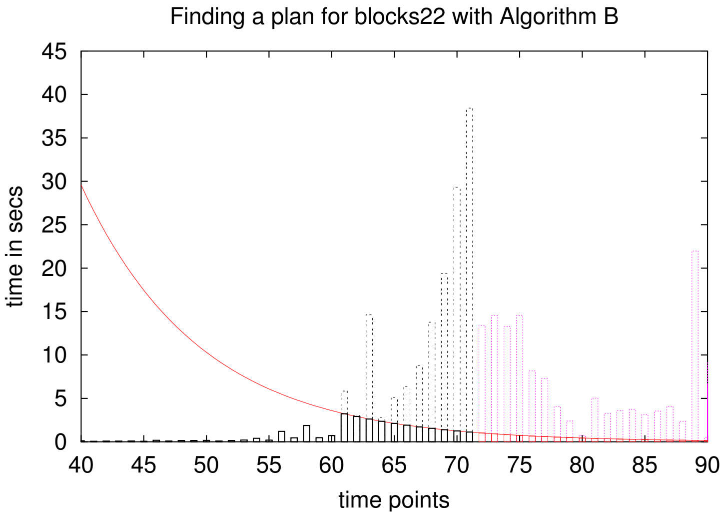
Geometric Evaluation



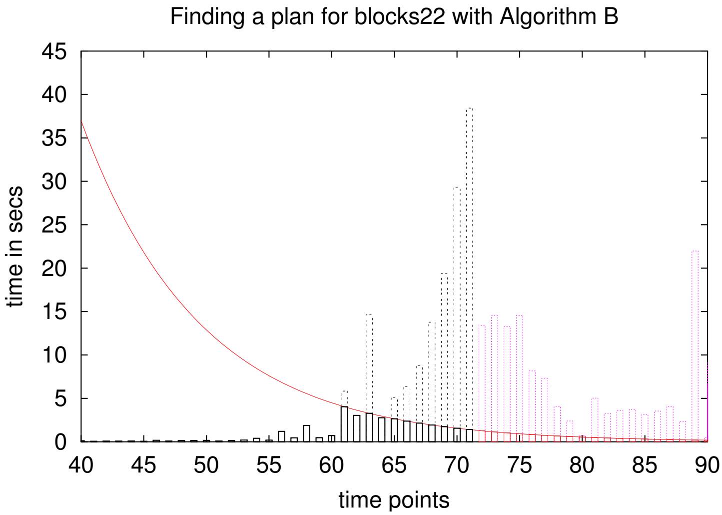
Geometric Evaluation



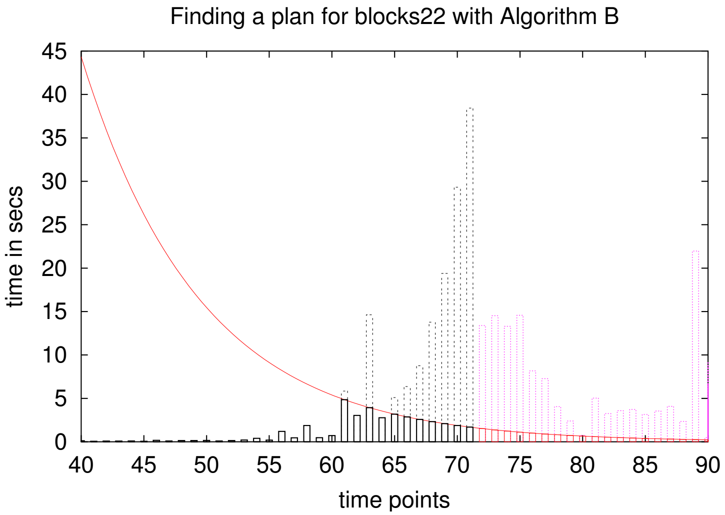
Geometric Evaluation



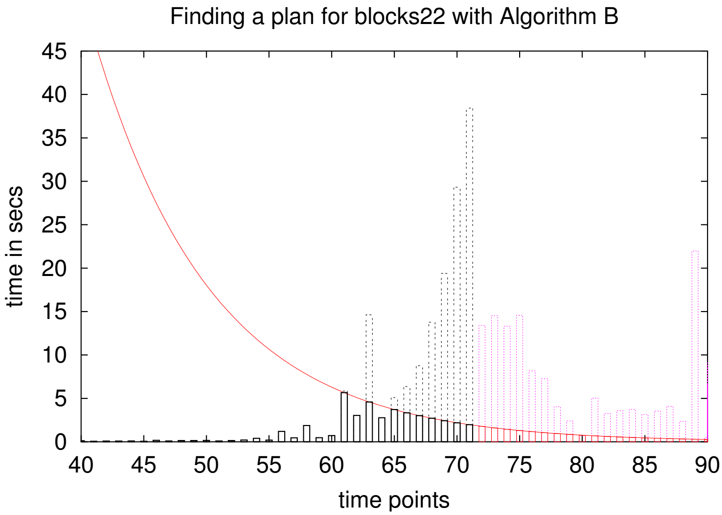
Geometric Evaluation



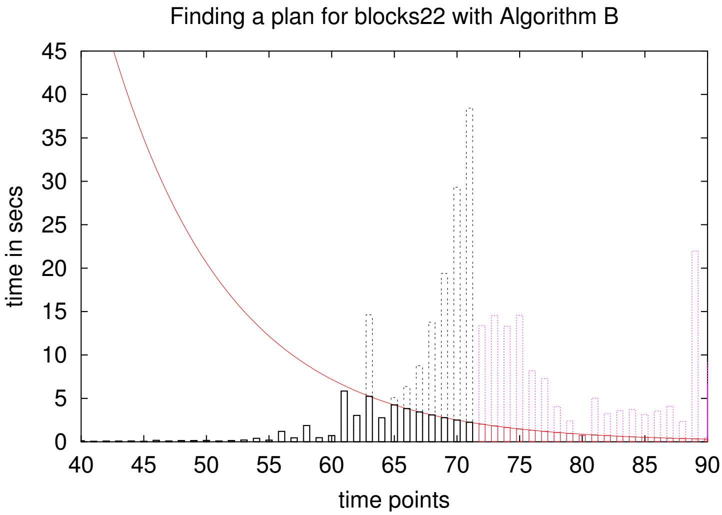
Geometric Evaluation



Geometric Evaluation



Geometric Evaluation



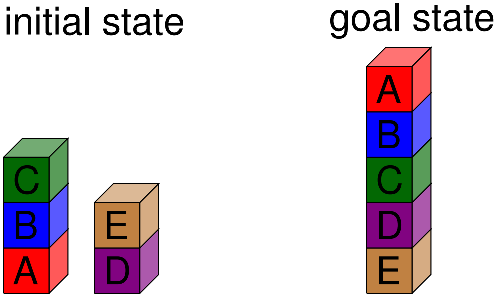
Solving the SAT Problem

SAT problems obtained from planning are solved by

- generic SAT solvers
    - Mostly based on Conflict-Driven Clause Learning (CDCL) [MMZ$^+$01].
    - Extremely good on hard combinatorial planning problems.
    - Not designed for solving the extremely large but “easy” formulas (arising in some types of benchmark problems).
- specialized SAT solvers [Rin10b, Rin10a]
    - Replace standard CDCL heuristics with planning-specific ones.
    - For certain problem classes substantial improvement
    - New research topic: lots of unexploited potential

Solving the SAT Problem

Example



Problem solved almost without search:

- Formulas for lengths 1 to 4 shown unsatisfiable without any search.
- Formula for plan length 5 is satisfiable: 3 nodes in the search tree.
- Plans have 5 to 7 operators, optimal plan has 5.

Solving the SAT Problem
Example

	0	1	2	3	4	5	0	1	2	3	4	5	0	1	2	3	4	5
clear(a)	F	F					F	F	F		T	T	F	F	F	T	T	T
clear(b)	F					F	F	F		T	T	F	F	F	T	T	T	F
clear(c)	T	T			F	F	T	T	T	T	F	F	T	T	T	T	F	F
clear(d)	F	T	T	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F
clear(e)	T	T	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F	F
on(a,b)	F	F	F			T	F	F	F	F	F	T	F	F	F	F	F	T
on(a,c)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(a,d)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(a,e)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(b,a)	T	T			F	F	T	T	T		F	F	T	T	T	F	F	F
on(b,c)	F	F			T	T	F	F	F	F	T	T	F	F	F	F	T	T
on(b,d)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(b,e)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(c,a)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(c,b)	T			F	F	F	T	T		F	F	F	T	T	F	F	F	F
on(c,d)	F	F	F	T	T	T	F	F	F	T	T	T	F	F	F	T	T	T
on(c,e)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,a)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,b)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,c)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,e)	F	F	T	T	T	T	F	F	T	T	T	T	F	F	T	T	T	T
on(e,a)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(e,b)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(e,c)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(e,d)	T	F	F	F	F	F	T	F	F	F	F	F	T	F	F	F	F	F
ontable(a)	T	T	T			F	T	T	T	T	T	F	T	T	T	T	T	F
ontable(b)	F	F			F	F	F	F	F		F	F	F	F	F	T	F	F
ontable(c)	F			F	F	F	F	F		F	F	F	F	F	T	F	F	F
ontable(d)	T	T	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F	F
ontable(e)	F	T	T	T	T	T	F	T	T	T	T	T	F	T	T	T	T	T

1. State variable values inferred from initial values and goals.
2. Branch: $\neg$clear(b)^1.
3. Branch: clear(a)^3.
4. Plan found:

	0	1	2	3	4
fromtable(a,b)	F	F	F	F	T
fromtable(b,c)	F	F	F	T	F
fromtable(c,d)	F	F	T	F	F
fromtable(d,e)	F	T	F	F	F
totable(b,a)	F	F	T	F	F
totable(c,b)	F	T	F	F	F
totable(e,d)	T	F	F	F	F

Solving the SAT Problem
Example

	0	1	2	3	4	5	0	1	2	3	4	5	0	1	2	3	4	5
clear(a)	F	F					F	F	F		T	T	F	F	F	T	T	T
clear(b)	F					F	F	F		T	T	F	F	F	T	T	T	F
clear(c)	T	T			F	F	T	T	T	T	F	F	T	T	T	T	F	F
clear(d)	F	T	T	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F
clear(e)	T	T	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F	F
on(a,b)	F	F	F			T	F	F	F	F	F	T	F	F	F	F	F	T
on(a,c)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(a,d)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(a,e)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(b,a)	T	T			F	F	T	T	T		F	F	T	T	T	F	F	F
on(b,c)	F	F			T	T	F	F	F	F	T	T	F	F	F	F	T	T
on(b,d)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(b,e)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(c,a)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(c,b)	T			F	F	F	T	T		F	F	F	T	T	F	F	F	F
on(c,d)	F	F	F	T	T	T	F	F	F	T	T	T	F	F	F	T	T	T
on(c,e)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,a)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,b)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,c)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,e)	F	F	T	T	T	T	F	F	T	T	T	T	F	F	T	T	T	T
on(e,a)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(e,b)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(e,c)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(e,d)	T	F	F	F	F	F	T	F	F	F	F	F	T	F	F	F	F	F
ontable(a)	T	T	T			F	T	T	T	T	T	F	T	T	T	T	T	F
ontable(b)	F	F			F	F	F	F	F		F	F	F	F	F	T	F	F
ontable(c)	F			F	F	F	F	F		F	F	F	F	F	T	F	F	F
ontable(d)	T	T	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F	F
ontable(e)	F	T	T	T	T	T	F	T	T	T	T	T	F	T	T	T	T	T

1. State variable values inferred from initial values and goals.
2. Branch: $\neg$clear(b)^1.
3. Branch: clear(a)^3.
4. Plan found:

	0	1	2	3	4
fromtable(a,b)	F	F	F	F	T
fromtable(b,c)	F	F	F	T	F
fromtable(c,d)	F	F	T	F	F
fromtable(d,e)	F	T	F	F	F
totable(b,a)	F	F	T	F	F
totable(c,b)	F	T	F	F	F
totable(e,d)	T	F	F	F	F

Solving the SAT Problem

Example

	0	1	2	3	4	5	0	1	2	3	4	5	0	1	2	3	4	5
clear(a)	F	F					F	F	F		T	T	F	F	F	T	T	T
clear(b)	F					F	F	F		T	T	F	F	F	T	T	T	F
clear(c)	T	T			F	F	T	T	T	T	F	F	T	T	T	T	F	F
clear(d)	F	T	T	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F
clear(e)	T	T	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F	F
on(a,b)	F	F	F			T	F	F	F	F	F	T	F	F	F	F	F	T
on(a,c)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(a,d)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(a,e)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(b,a)	T	T			F	F	T	T	T		F	F	T	T	T	F	F	F
on(b,c)	F	F			T	T	F	F	F	F	T	T	F	F	F	F	T	T
on(b,d)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(b,e)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(c,a)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(c,b)	T			F	F	F	T	T		F	F	F	T	T	F	F	F	F
on(c,d)	F	F	F	T	T	T	F	F	F	T	T	T	F	F	F	T	T	T
on(c,e)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,a)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,b)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,c)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,e)	F	F	T	T	T	T	F	F	T	T	T	T	F	F	T	T	T	T
on(e,a)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(e,b)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(e,c)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(e,d)	T	F	F	F	F	F	T	F	F	F	F	F	T	F	F	F	F	F
ontable(a)	T	T	T			F	T	T	T	T	T	F	T	T	T	T	T	F
ontable(b)	F	F			F	F	F	F	F		F	F	F	F	F	T	F	F
ontable(c)	F			F	F	F	F	F		F	F	F	F	F	T	F	F	F
ontable(d)	T	T	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F	F
ontable(e)	F	T	T	T	T	T	F	T	T	T	T	T	F	T	T	T	T	T

1. State variable values inferred from initial values and goals.
2. Branch: $\neg$clear(b)^1.
3. Branch: clear(a)^3.
4. Plan found:

	0	1	2	3	4
fromtable(a,b)	F	F	F	F	T
fromtable(b,c)	F	F	F	T	F
fromtable(c,d)	F	F	T	F	F
fromtable(d,e)	F	T	F	F	F
totable(b,a)	F	F	T	F	F
totable(c,b)	F	T	F	F	F
totable(e,d)	T	F	F	F	F

Solving the SAT Problem

Example

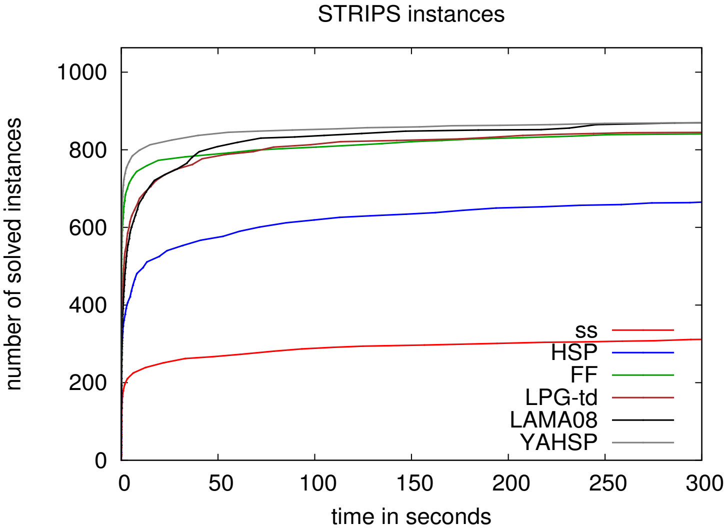
	0	1	2	3	4	5	0	1	2	3	4	5	0	1	2	3	4	5
clear(a)	F	F					F	F	F		T	T	F	F	F	T	T	T
clear(b)	F					F	F	F		T	T	F	F	F	T	T	T	F
clear(c)	T	T			F	F	T	T	T	T	F	F	T	T	T	T	F	F
clear(d)	F	T	T	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F
clear(e)	T	T	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F	F
on(a,b)	F	F	F			T	F	F	F	F	F	T	F	F	F	F	F	T
on(a,c)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(a,d)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(a,e)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(b,a)	T	T			F	F	T	T	T		F	F	T	T	T	F	F	F
on(b,c)	F	F			T	T	F	F	F	F	T	T	F	F	F	F	T	T
on(b,d)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(b,e)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(c,a)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(c,b)	T			F	F	F	T	T		F	F	F	T	T	F	F	F	F
on(c,d)	F	F	F	T	T	T	F	F	F	T	T	T	F	F	F	T	T	T
on(c,e)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,a)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,b)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,c)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,e)	F	F	T	T	T	T	F	F	T	T	T	T	F	F	T	T	T	T
on(e,a)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(e,b)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(e,c)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(e,d)	T	F	F	F	F	F	T	F	F	F	F	F	T	F	F	F	F	F
ontable(a)	T	T	T			F	T	T	T	T	T	F	T	T	T	T	T	F
ontable(b)	F	F			F	F	F	F	F		F	F	F	F	F	T	F	F
ontable(c)	F			F	F	F	F	F		F	F	F	F	F	T	F	F	F
ontable(d)	T	T	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F	F
ontable(e)	F	T	T	T	T	T	F	T	T	T	T	T	F	T	T	T	T	T

1. State variable values inferred from initial values and goals.
2. Branch: $\neg$clear(b)^1.
3. Branch: clear(a)^3.
4. Plan found:

	0	1	2	3	4
fromtable(a,b)	F	F	F	F	T
fromtable(b,c)	F	F	F	T	F
fromtable(c,d)	F	F	T	F	F
fromtable(d,e)	F	T	F	F	F
totable(b,a)	F	F	T	F	F
totable(c,b)	F	T	F	F	F
totable(e,d)	T	F	F	F	F

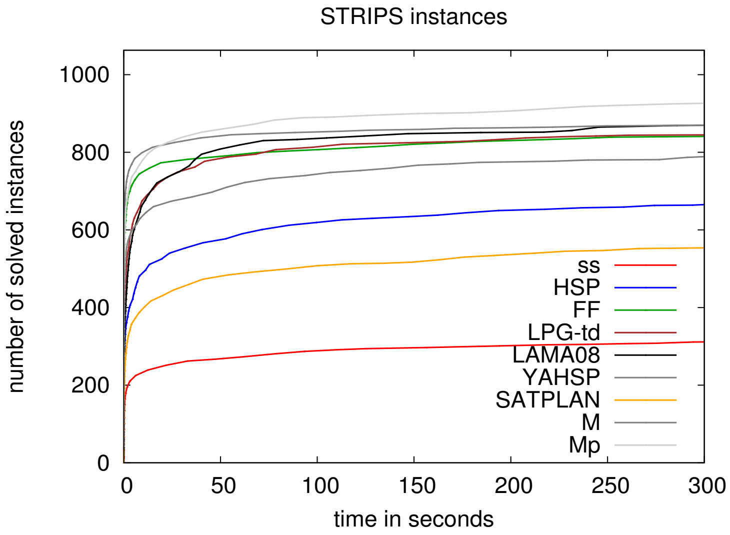
Performance of SAT-Based Planners

Planning Competition Problems 1998-2008



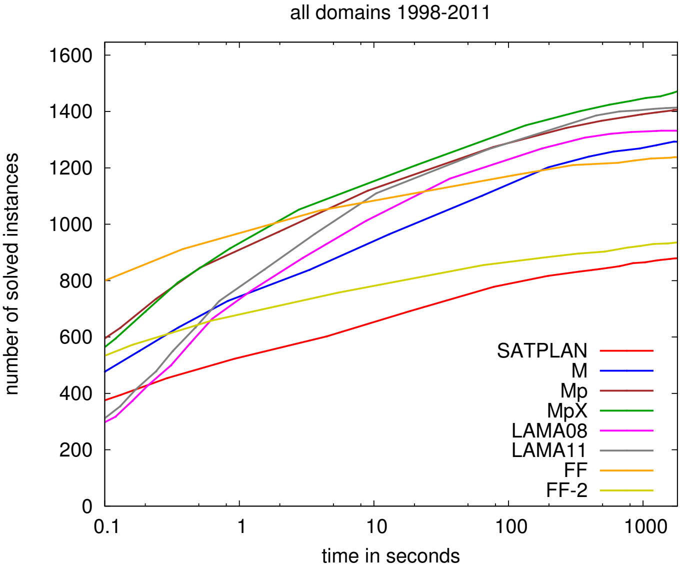
Performance of SAT-Based Planners

Planning Competition Problems 1998-2008



Performance of SAT-Based Planners

Planning Competition Problems 1998-2011 (revised)



Extensions

MathSAT [BBC$^+$05] and other SAT modulo Theories (SMT) solvers extend SAT with numerical variables and equalities and inequalities.
Applications include:

- timed systems [ACKS02], temporal planning
- hybrid systems [GPB05, ABCS05], temporal planning + continuous change

Symbolic Search Methods

Motivation

- logical formulas as a data structure for sets, relations
- Planning (model-checking, diagnosis, ...) algorithms in terms of set & relational operations.
- Algorithms that can handle very large state sets efficiently, bypassing inherent limitations of explicit state-space search.
- Complementary to explicit (enumerative) representations of state sets: strengths in different types of problems.

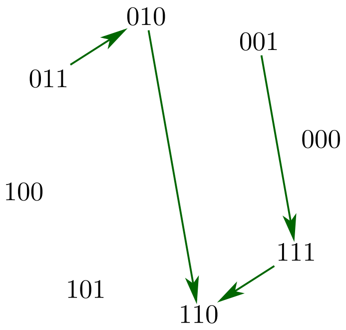
Transition relations in propositional logic

State variables are
$X = \{a, b, c\}$.

$$
\begin{array}{l}
(\neg a \wedge b \wedge c \wedge \neg a' \wedge b' \wedge \neg c')\vee \\
(\neg a \wedge b \wedge \neg c \wedge a' \wedge b' \wedge \neg c')\vee \\
(\neg a \wedge \neg b \wedge c \wedge a' \wedge b' \wedge c')\vee \\
(a \wedge b \wedge c \wedge a' \wedge b' \wedge \neg c')
\end{array}
$$

The corresponding matrix is

$$
\begin{array}{c|cccccccc}
 & 000 & 001 & 010 & 011 & 100 & 101 & 110 & 111 \\
\hline
000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
001 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
010 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
011 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
100 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
101 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
110 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
111 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{array}
$$



Operations

The image of a set T of states w.r.t. action a is

$$\textit{img}_a(T) = \{s' \in S | s \in T, sas'\}.$$

The pre-image of a set T of states w.r.t. action a is

$$\textit{preimg}_a(T) = \{s \in S | s' \in T, sas'\}.$$

These operations reduce to the relational join and projection operations with a logic-representation of sets (unary relations) and binary relations.

Finding Plans with a Symbolic Algorithm

Computation of all reachable states

$$S_0 = \{I\}$$
$$S_{i+1} = S_i \cup \bigcup_{x \in X} \mathit{img}_x(S_i)$$

If $S_i = S_{i+1}$, then $S_j = S_i$ for all $j \geq i$, and the computation can be terminated.

- $S_i, i \geq 0$ is the set of states with distance $\leq i$ from the initial state.
- $S_i \backslash S_{i-1}, i \geq 1$ is the set of states with distance i.
- If $G \cap S_i$ for some $i \geq 0$, then there is a plan.

Action sequence recovered from sets S_i by a sequence of backward-chaining steps.

Use in Connection with Heuristic Search Algorithms

Symbolic (BDD) versions of heuristic algorithms in the state-space search context:

- SetA* [JVB08]
- BDDA* [ER98]
- ADDA* [HZF02]

Use in Connection with More General Problems

- BDDs and other normal forms standard representation in planning with partial observability [BCRT01, Rin05]. Also, probabilistic planning [HSAHB99] with value functions represented as Algebraic Decision Diagrams (ADD) [FMY97, BFG$^+$97].
- A belief state is a set of possible current states.
- These sets are often very large, best represented as formulas.

Significance of Symbolic Representations

- Much more powerful framework than SAT or explicit state-space search.
- Unlike other methods, allows exhaustive generation of reachable states.
- Problem 1: e.g. with BDDs, size of transition relation may explode.
- Problem 2: e.g. with BDDs, size of sets S_i may explode.
- Important research topic: symbolic search with less restrictive normal forms than BDD.

Images as Relational Operations

$$\begin{array}{|c|}\hline s_0 \\ s_2 \\ \hline\end{array} \bowtie \begin{array}{|cc|}\hline s_0 & s_1 \\ s_0 & s_2 \\ s_1 & s_0 \\ s_1 & s_2 \\ s_2 & s_0 \\ \hline\end{array} = \begin{array}{|cc|}\hline s_0 & \mathbf{s_1} \\ s_0 & \mathbf{s_2} \\ s_2 & \mathbf{s_0} \\ \hline\end{array}$$

Images as Relational Operations

$$\boxed{\begin{array}{l} s_0 \\ s_2 \end{array}} \bowtie \boxed{\begin{array}{ll} s_0 & s_1 \\ s_0 & s_2 \\ s_1 & s_0 \\ s_1 & s_2 \\ s_2 & s_0 \end{array}} = \boxed{\begin{array}{ll} s_0 & \mathbf{s_1} \\ s_0 & \mathbf{s_2} \\ s_2 & \mathbf{s_0} \end{array}}$$

$$\boxed{\begin{array}{l} s_0 00 \\ s_2 10 \end{array}} \bowtie \boxed{\begin{array}{l} s_0 s_1 00\ 01 \\ s_0 s_2 00\ 10 \\ s_1 s_0 01\ 00 \\ s_1 s_2 01\ 10 \\ s_2 s_0 10\ 00 \end{array}} = \boxed{\begin{array}{l} s_0 s_1 00\ 01 \\ s_0 s_2 00\ 10 \\ s_2 s_0 10\ 00 \end{array}}$$

Images as Relational Operations

$$\boxed{\begin{array}{l} s_0 00 \\ s_2 10 \end{array}} \bowtie \boxed{\begin{array}{l} s_0 s_1 00\ 01 \\ s_0 s_2 00\ 10 \\ s_1 s_0 01\ 00 \\ s_1 s_2 01\ 10 \\ s_2 s_0 10\ 00 \end{array}} = \boxed{\begin{array}{l} s_0 s_1 00\ 01 \\ s_0 s_2 00\ 10 \\ s_2 s_0 10\ 00 \end{array}}$$

$$\begin{array}{l|l} x_0 x_1 & \\ \hline 00 & 1 \\ 01 & 0 \\ 10 & 1 \\ 11 & 0 \end{array} \bowtie \begin{array}{l|l} x_0 x_1 x_0' x_1' & \\ \hline 0000 & 0 \\ 0001 & 1 \\ 0010 & 1 \\ 0011 & 0 \\ 0100 & 1 \\ 0101 & 0 \\ 0110 & 1 \\ 0111 & 0 \\ 1000 & 1 \\ 1001 & 0 \\ 1010 & 0 \\ 1011 & 0 \\ 1100 & 0 \\ 1101 & 0 \\ 1110 & 0 \\ 1111 & 0 \end{array} = \begin{array}{l|l} x_0 x_1 x_0' x_1' & \\ \hline 0001 & 1 \\ 0010 & 1 \\ 1000 & 1 \end{array}$$

Representation of Sets as Formulas

state sets	**formulas over X**
those $\frac{2^{\lvert X \rvert}}{2}$ states where x is true	$x \in X$
$\overline{E}$ (complement)	$\neg E$
$E \cup F$	$E \vee F$
$E \cap F$	$E \wedge F$
$E \setminus F$ (set difference)	$E \wedge \neg F$
the empty set $\emptyset$	$\perp$ (constant *false*)
the universal set	$\top$ (constant *true*)

question about sets	**question about formulas**
$E \subseteq F$?	$E \models F$?
$E \subset F$?	$E \models F$ and $F \not\models E$?
$E = F$?	$E \models F$ and $F \models E$?

Sets (of states) as formulas

Formulas over X represent sets

$a \vee b$ over $X = \{a, b, c\}$

represents the set $\{\overset{a\,b\,c}{010}, 011, 100, 101, 110, 111\}$.

Formulas over $X \cup X'$ represent binary relations

$a \wedge a' \wedge (b \leftrightarrow b')$ over $X \cup X'$ where $X = \{a, b\}$, $X' = \{a', b'\}$

represents the binary relation $\{(\overset{a\,b}{10}, \overset{a'b'}{1\,0}), (11, 11)\}$.

Valuations $\overset{a\,b\,a'b'}{1010}$ and 1111 of $X \cup X'$ can be viewed respectively as pairs of valuations $(\overset{a\,b}{10}, \overset{a'b'}{1\,0})$ and $(11, 11)$ of X.

Relation Operations

relation operation	logical operation
projection	abstraction
join	conjunction

Normal Forms

normal form	**reference**	**comment**
NNF Negation Normal Form		
DNF Disjunctive Normal Form		
CNF Conjunctive Normal Form		
BDD Binary Decision Diagram	[Bry92]	most popular
DNNF Decomposable NNF	[Dar01]	more compact

Darwiche's terminology: knowledge compilation languages [DM02]

Trade-off

- more compact $\mapsto$ less efficient operations
- But, "more efficient" is in the size of a correspondingly inflated formula. (Also more efficient in terms of wall clock?) BDD-SAT is $\mathcal{O}(1)$, but e.g. translation into BDDs is (usually) far less efficient than testing SAT directly.

Complexity of Operations

Operations offered e.g. by BDD packages:

	$\vee$	$\wedge$	$\neg$	$\phi \in$TAUT?	$\phi \in$SAT?	$\phi \equiv \phi'$?
NNF	poly	poly	poly	co-NP-hard	NP-hard	co-NP-hard
DNF	poly	exp	exp	co-NP-hard	in P	co-NP-hard
CNF	exp	poly	exp	in P	NP-hard	co-NP-hard
BDD	exp	exp	poly	in P	in P	in P

Remark

For BDDs one $\vee/\wedge$ *is polynomial time/size (size is doubled) but repeated* $\vee/\wedge$ *lead to exponential size.*

Existential and Universal Abstraction

Definition

Existential abstraction of a formula ϕ with respect to $x \in X$:

$$\exists x.\phi = \phi[\top/x] \vee \phi[\bot/x].$$

Universal abstraction is defined analogously by using conjunction instead of disjunction.

Definition

Universal abstraction of a formula ϕ with respect to $x \in X$:

$$\forall x.\phi = \phi[\top/x] \wedge \phi[\bot/x].$$

∃-Abstraction

Example

$$\begin{aligned}
&\exists b.((a \rightarrow b) \wedge (b \rightarrow c)) \\
&= ((a \rightarrow \top) \wedge (\top \rightarrow c)) \vee ((a \rightarrow \bot) \wedge (\bot \rightarrow c)) \\
&\equiv c \vee \neg a \\
&\equiv a \rightarrow c
\end{aligned}$$

$$\begin{aligned}
&\exists ab.(a \vee b) = \exists b.(\top \vee b) \vee (\bot \vee b) \\
&= ((\top \vee \top) \vee (\bot \vee \top)) \vee ((\top \vee \bot) \vee (\bot \vee \bot)) \\
&\equiv (\top \vee \top) \vee (\top \vee \bot) \equiv \top
\end{aligned}$$

$\forall$ and $\exists$-Abstraction in Terms of Truth-Tables

$\forall c$ and $\exists c$ correspond to combining lines with the same valuation for variables other than c.

Example

$$\exists c.(a \lor (b \land c)) \equiv a \lor b \qquad \forall c.(a \lor (b \land c)) \equiv a$$

a	b	c	$a \lor (b \land c)$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

a	b	$\exists c.(a \lor (b \land c))$
0	0	0
0	1	1
1	0	1
1	1	1

a	b	$\forall c.(a \lor (b \land c))$
0	0	0
0	1	0
1	0	1
1	1	1

Encoding of Actions as Formulas

Let X be the set of all state variables. An action a corresponds to the conjunction of the precondition P_j and

$$x' \leftrightarrow F_i(X)$$

for all $x \in X$. Denote this by $\tau_X(a)$.

Example (move-from-A-to-B)

$$atA \wedge (atA' \leftrightarrow \bot) \wedge (atB' \leftrightarrow \top) \wedge (atC' \leftrightarrow atC) \wedge (atD' \leftrightarrow atD)$$

This is exactly the same as in the SAT case, except that we have x and x' instead of $x@t$ and $x@(t+1)$.

Computation of Successor States

Let

- $X = \{x_1, \dots, x_n\}$,
- $X' = \{x'_1, \dots, x'_n\}$,
- ϕ be a formula over X that represents a set T of states.

Image Operation

The image $\{s' \in S | s \in T, sas'\}$ of T with respect to a is

$$\mathit{img}_a(\phi) = (\exists X.(\phi \wedge \tau_X(a)))[X/X'].$$

The renaming is necessary to obtain a formula over X.

Computation of Predecessor States

Let

- $X = \{x_1, \dots, x_n\}$,
- $X' = \{x'_1, \dots, x'_n\}$,
- ϕ be a formula over X that represents a set T of states.

Preimage Operation

The pre-image $\{s \in S | s' \in T, sas'\}$ of T with respect to a is

$$\textit{preimg}_a(\phi) = (\exists X'.(\phi[X'/X] \wedge \tau_X(a))).$$

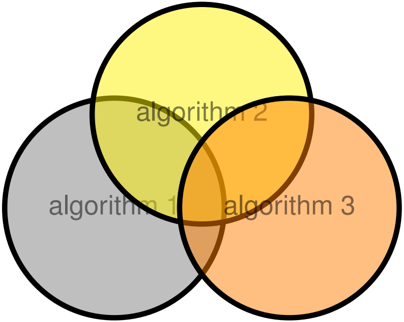
The renaming of ϕ is necessary so that we can start with a formula over X.

Engineering Efficient Planners

- Gap between Theory and Practice large: engineering details of implementation critical for performance in current planners.
- Few of the most efficient planners use textbook methods.
- Explanations for the observed differences between planners lacking: this is more art than science.

Algorithm Portfolios

- Algorithm portfolio = combination of two or more algorithms
- Useful if there is no single “strongest” algorithm.



Algorithm Portfolios
Composition methods

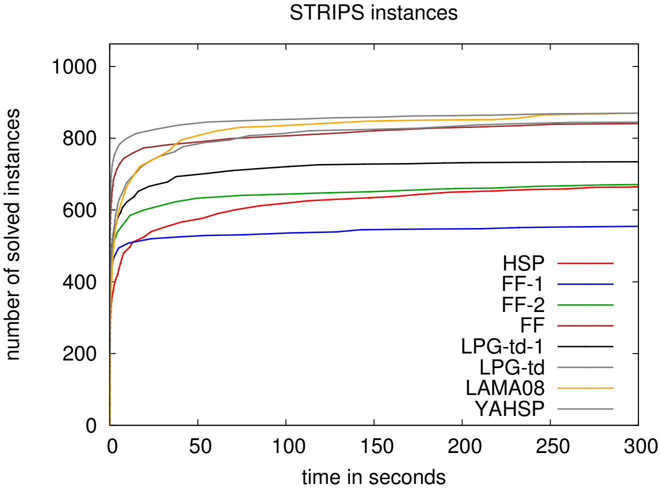
Composition methods:

- selection = choose one, for the instance in question
- parallel composition = run components in parallel
- sequential composition = run consecutively, according to a schedule

Examples: BLACKBOX [KS99], FF [HN01], LPG [GS02] (all use sequential composition)

Algorithm Portfolios

An Illustration of Portfolios



FF	=	FF-1 followed by FF-2
LPG-td	=	LPGT-td-1 followed by FF-2

Evaluation of Planners

Evaluation of planning systems is based on

- Hand-crafted problems (from the planning competitions)
  - This is the most popular option.
  - \+ Problems with (at least moderately) different structure.
  - \- Real-world relevance mostly low.
  - \- Instance generation uncontrolled: not known if easy or difficult.
  - \- Many have a similar structure: objects moving in a network.
- Benchmark sets obtained by translation from other problems
  - graph-theoretic problems: cliques, colorability, ... [PMB11]
- Instances sampled from all instances [Byl96].
  - \+ Easy to control problem hardness.
  - \- No direct real-world relevance (but: core of any “hard” problem)

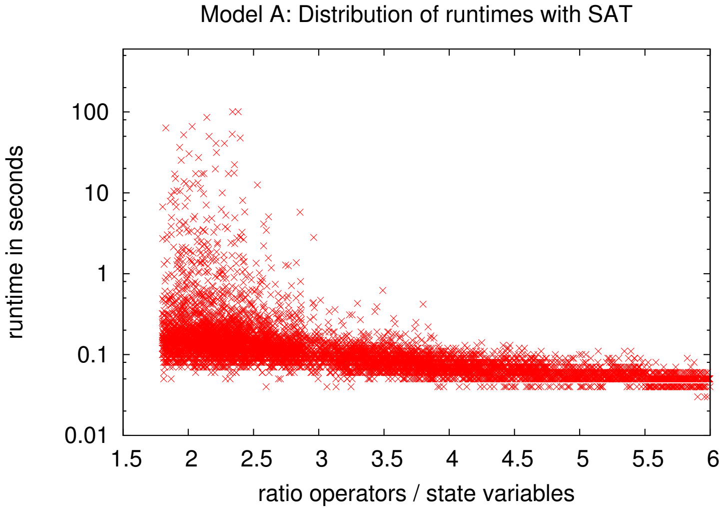
Sampling from the Set of All Instances

[Byl96, Rin04c]

- Generation:
  1. Fix number N of state variables, number M of actions.
  2. For each action, choose preconditions and effects randomly.
- Has a phase transition from unsolvable to solvable, similarly to SAT [MSL92] and connectivity of random graphs [Bol85].
- Exhibits an easy-hard-easy pattern, for a fixed N and an increasing M, analogously to SAT [MSL92].
- Hard instances roughly at the 50 per cent solvability point.
- Hardest instances are very hard: 20 state variables too difficult for many planners, as their heuristics don't help.

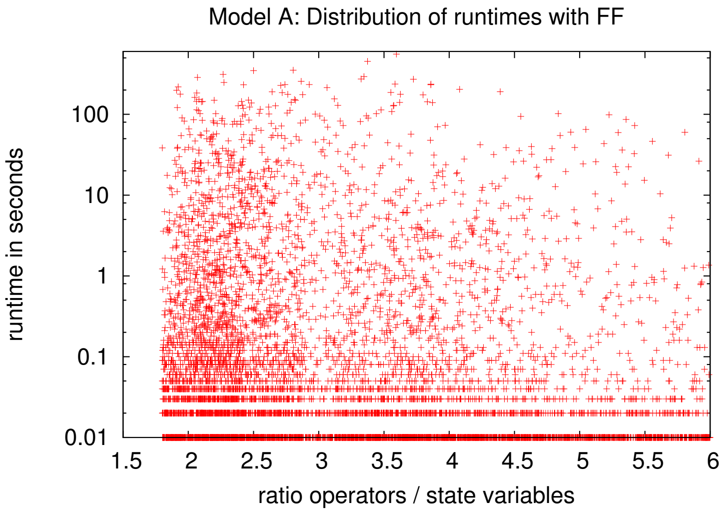
Sampling from the Set of All Instances

Experiments with planners



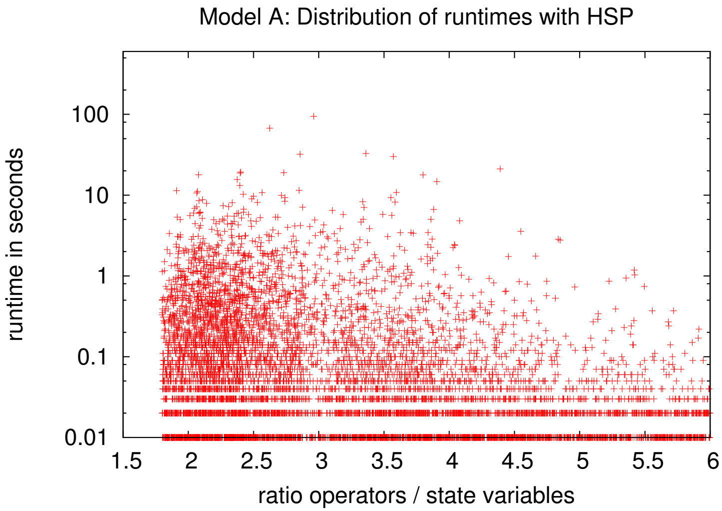
Sampling from the Set of All Instances

Experiments with planners



Sampling from the Set of All Instances

Experiments with planners



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